REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
24 November 2015	Briefing Charts	02 Nov 2015 – 24 Nov 2015

4. TITLE AND SUBTITLE
How Many Grid Points are Required for Time Accurate Simulations?
Scheme Selection and Scale-Discriminant Stabilization

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
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5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER
Q12J

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) AND ADDRESS(ES)

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8. PERFORMING ORGANIZATION REPORT NO.

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory (AFMC)
AFRL/RQR
5 Pollux Drive
Edwards AFB, CA 93524-7048

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-RQ-ED-VG-2015-410

12. DISTRIBUTION / AVAILABILITY STATEMENT
Distribution approved for public Release; distribution unlimited

13. SUPPLEMENTARY NOTES
For presentation at 2015 APS/Division of Fluid Dynamics; Boston, MA (24 November 2015)
PA Clearance Number 15671, Clearance Date 11/10/2015

14. ABSTRACT
Briefing Charts/Viewgraphs

15. SUBJECT TERMS
N/A

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					V. Sankaran
a. REPORT	b. ABSTRACT	c. THIS PAGE	SAR	16	19b. TELEPHONE NO *(include area code)*
Unclassified	**Unclassified**	**Unclassified**			N/A

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. 239.18

How Many Grid Points are Required for Time Accurate Simulations?

Scheme Selection and Scale-Discriminant Stabilization

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Research Supported by AFOSR (Drs. Fahroo and Li, PMs)

APS DSD 2015
Boston, MA

ENERGY & PROPULSION RESEARCH LABORATORY

Motivation

- computational efficiency
  - coarser grids
  - larger time steps

<u>spatial concerns:</u>

- how well are gradients captured? (resolution requirement)

<u>spatial/temporal concerns:</u>

- dispersion and dissipation error
  - better characteristics for a broader range of wavenumbers

<u>scheme stabilization concerns:</u>

- balancing accuracy with stability
  - artificial dissipation, filtering

Spectral Representation

Fourier basis:

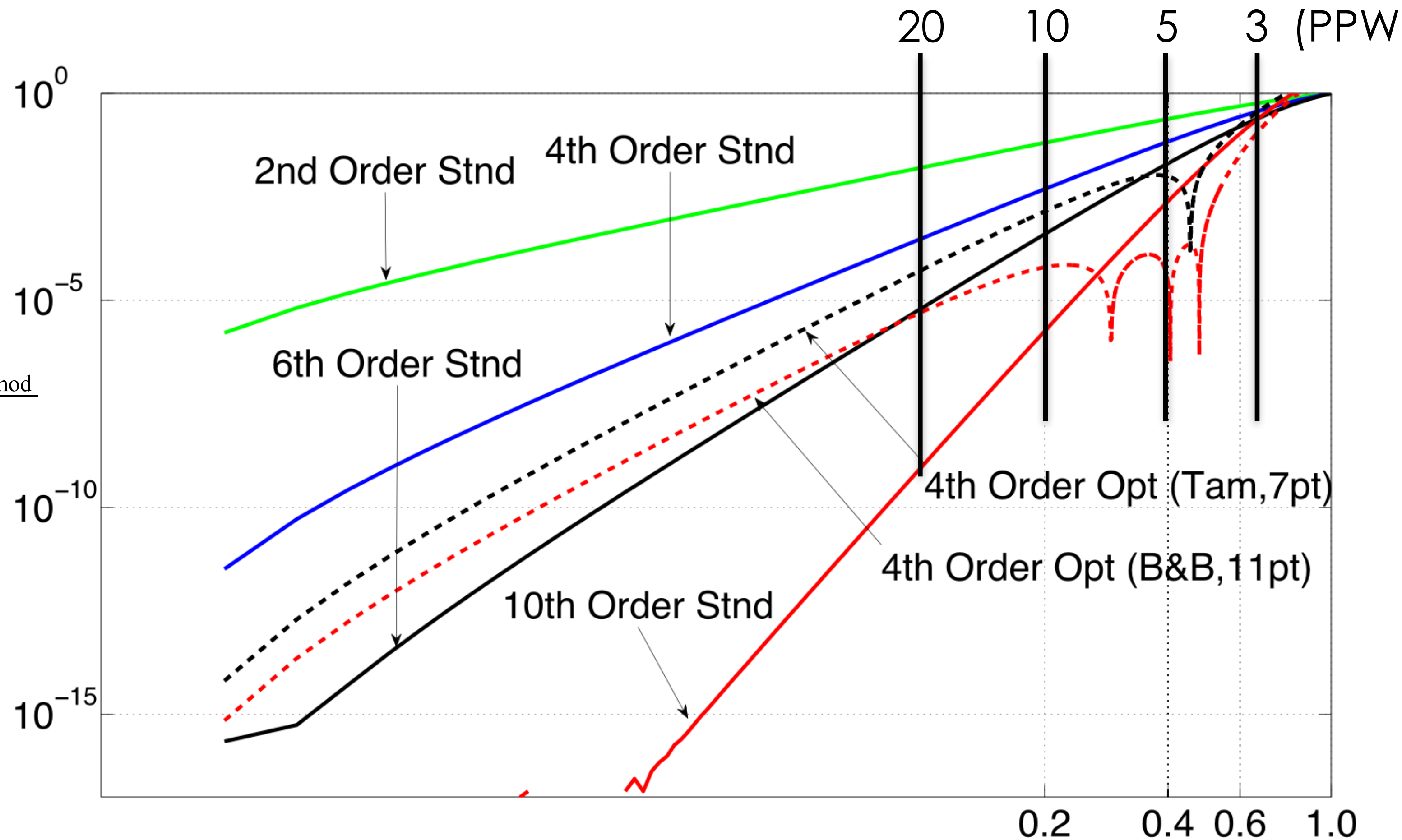
$$
\begin{aligned}
u_j &= \sum_k \hat{u}(k) e^{ikx_j} \\
u_{j+1} &= \sum_k \hat{u}(k) e^{ikx_j} e^{ik\Delta x} \quad \text{with} \quad \theta = (k\Delta x) \in [-\pi, \pi]
\end{aligned}
$$

$$\delta_x u \rightarrow a_o u'_j + \sum_{l=1} a_l \left(u'_{j+l} - u'_{j-l} \right) = \frac{1}{\Delta x}\left[b_o u_j + \sum_{r=1} b_r \left(u_{j+r} - u_{j-r} \right) \right]$$

$$Z_{conv,spec} = i \circlearrowright k \qquad \rightarrow Z_{conv}(\theta) = \underbrace{b_o + i \circlearrowright \left[\frac{2\sum_{r=1}^{R} b_r \sin(r\theta)}{1 + 2\sum_{l=1}^{L} a_l \sin(l\theta)} \right]}_{\text{modified wavenumber}}$$

Modified Wavenumber

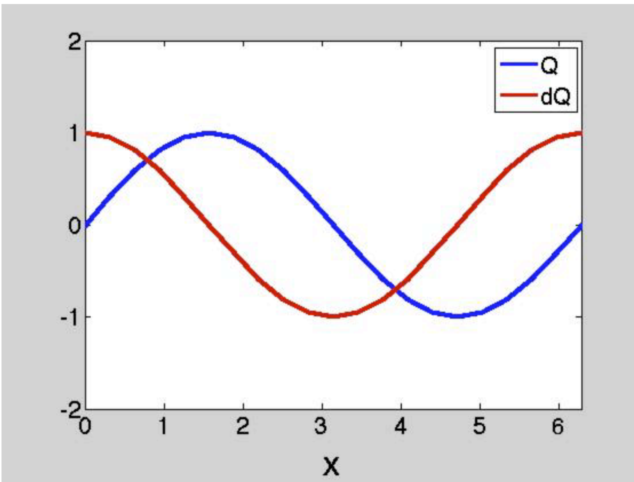
$$\text{error}_k = \frac{k - k_{\text{mod}}}{k}$$



goal: proper representation of derivative

Gradient Capture vs. Resolution: Single Mode

Solution/Derivative:



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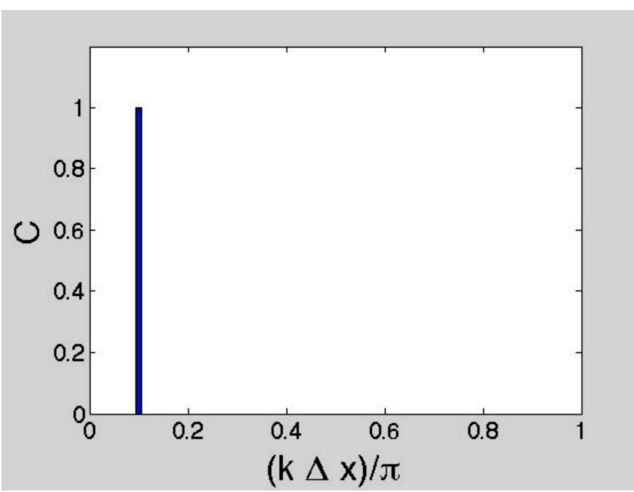
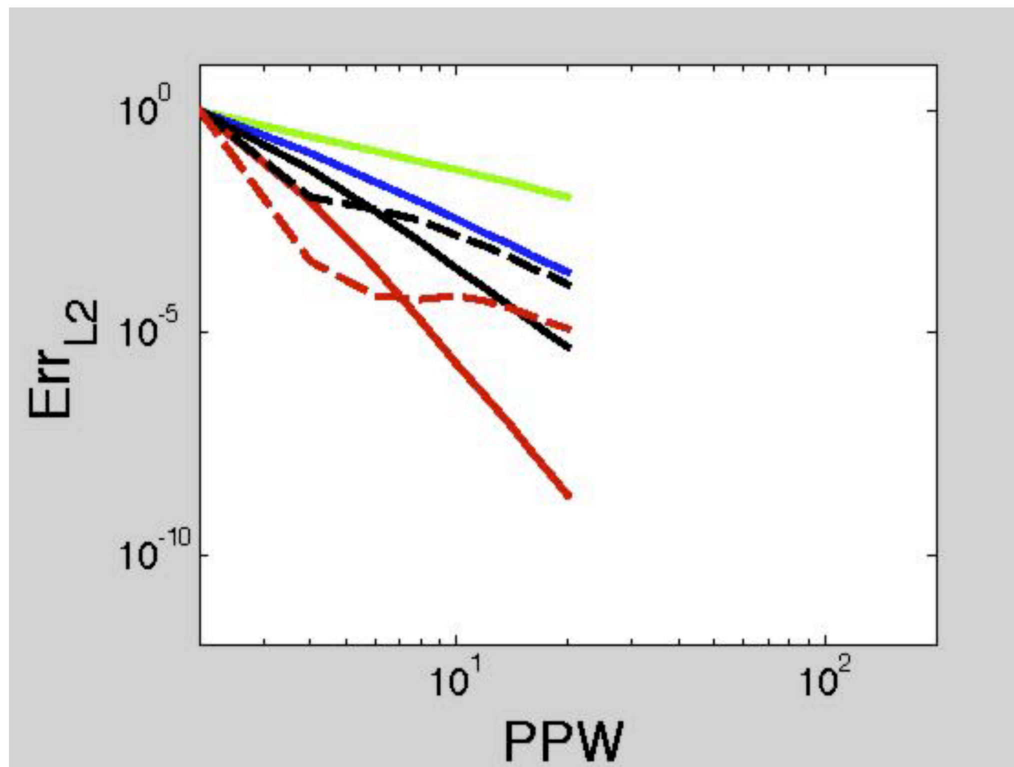


$$f(x) = \sin(x) \quad \text{with } x \in [0, 2\pi]$$

$$\frac{df}{dx} = \cos(x)$$

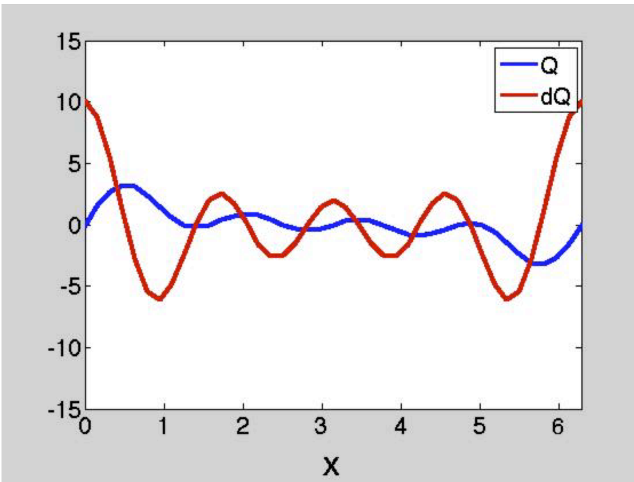
Convergence:

__ CD02
__ CD04
__ CD06
-- CD04 (Tam,7pt)
__ CD10
-- CD04 (B&B,11pt)



Gradient Capture vs. Resolution: Multiple Modes

Solution/Derivative:



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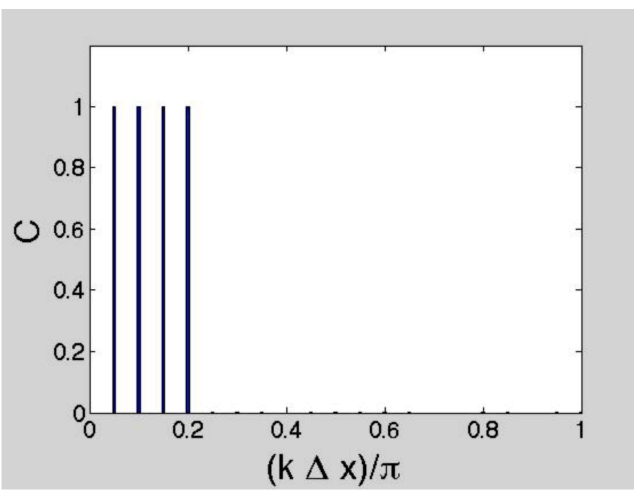
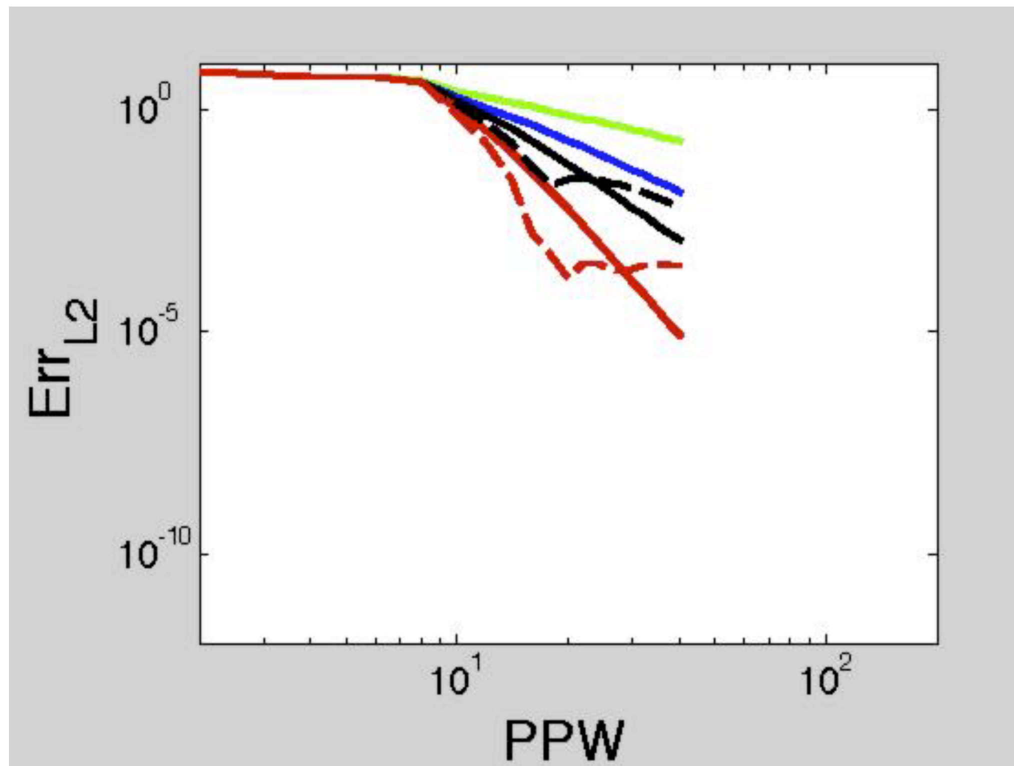


$$f(x) = \sum_{m=1}^{4} \sin(mx) \quad \text{with } x \in [0, 2\pi]$$

$$\frac{df}{dx} = \sum_{m=1}^{4} m\cos(mx)$$

Convergence:

- __ CD02
- __ CD04
- __ CD06
- -- CD04 (Tam,7pt)
- __ CD10
- -- CD04 (B&B,11pt)

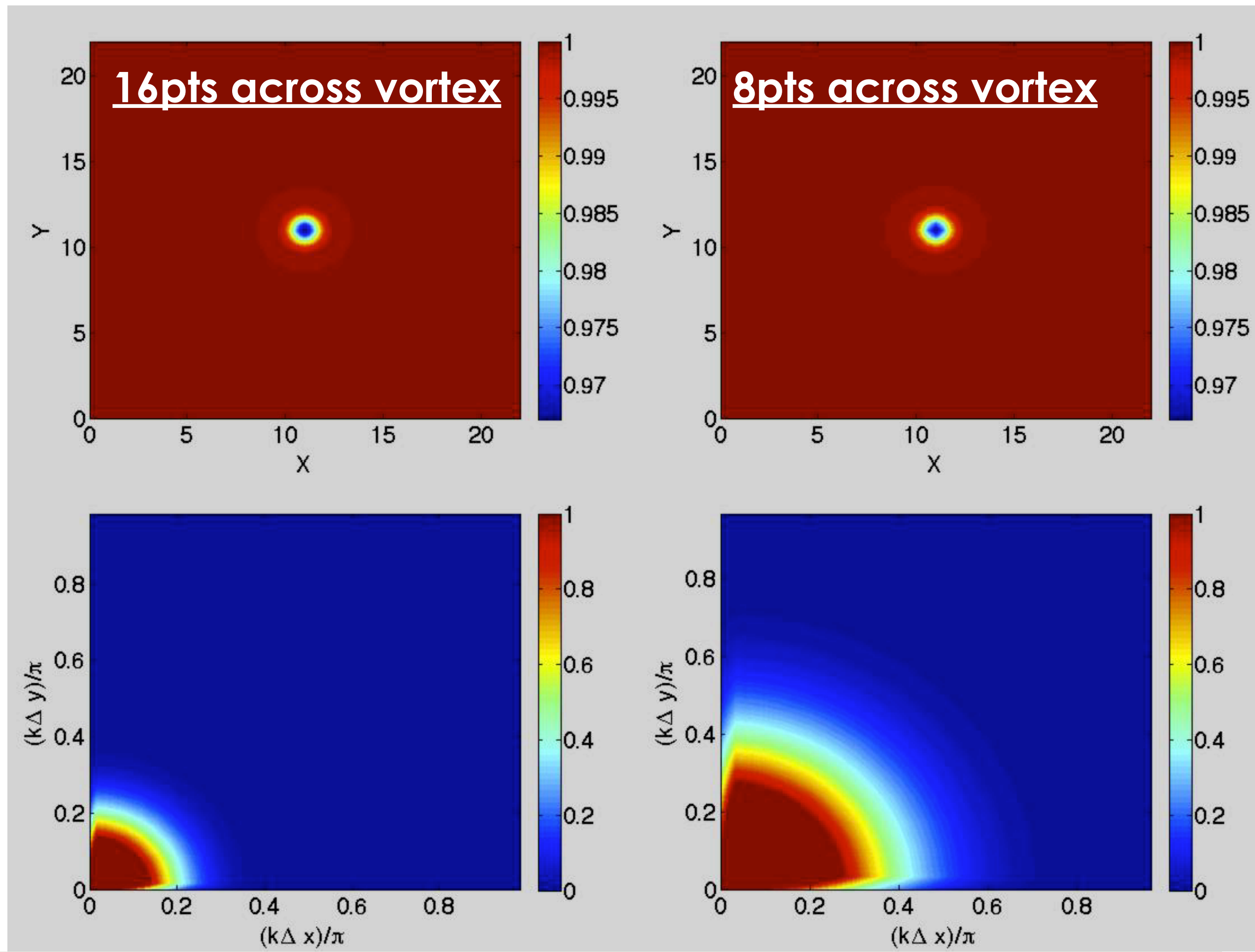


Isentropic Vortex: no Stabilization

$M_\infty = 0.5$

$CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1$

CD04 (11pt B&B)



need to stabilize accumulation of high frequency error

Stabilization:
Artificial Dissipation and Filtering

Artificial Dissipation:

$$\frac{\partial Q}{\partial t} = -\frac{\partial E}{\partial x} + \sum_m (-1)^{m-1} (\Delta x)^{2m-1} \varepsilon_{2m} |\lambda_{u+c}| \frac{\partial^{2m} Q}{\partial x^{2m}}$$

damping strongly dependent on base scheme (couples with temporal scheme)

Filtering:

$$\frac{\partial Q^*}{\partial t} = -\frac{\partial E}{\partial x}$$

$$\left[1 + \sum_m^M (-1)^{m+1} \varepsilon_{IF,2m} \left(\frac{\Delta x}{2}\right)^{2m} \frac{\partial^{2m}}{\partial x^{2m}}\right] Q = \left[1 + \sum_n^N (-1)^{n+1} \varepsilon_{EF,2n} \left(\frac{\Delta x}{2}\right)^{2n} \frac{\partial^{2n}}{\partial x^{2n}}\right] Q^*$$

adds consistent amount of damping to base scheme (decoupled from temporal integration)

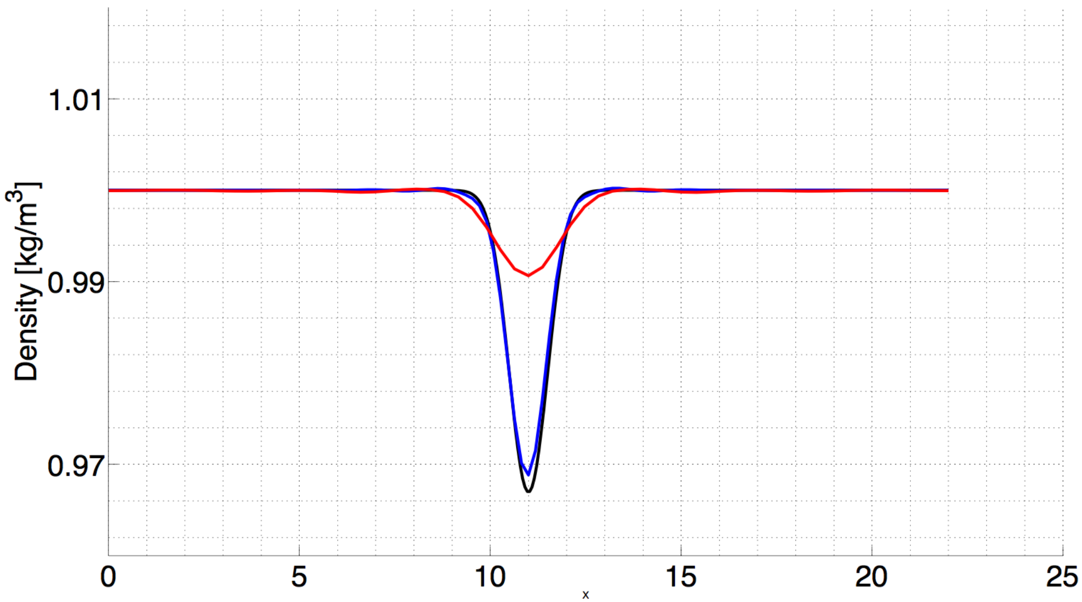
Isentropic Vortex: Traditional Stabilization

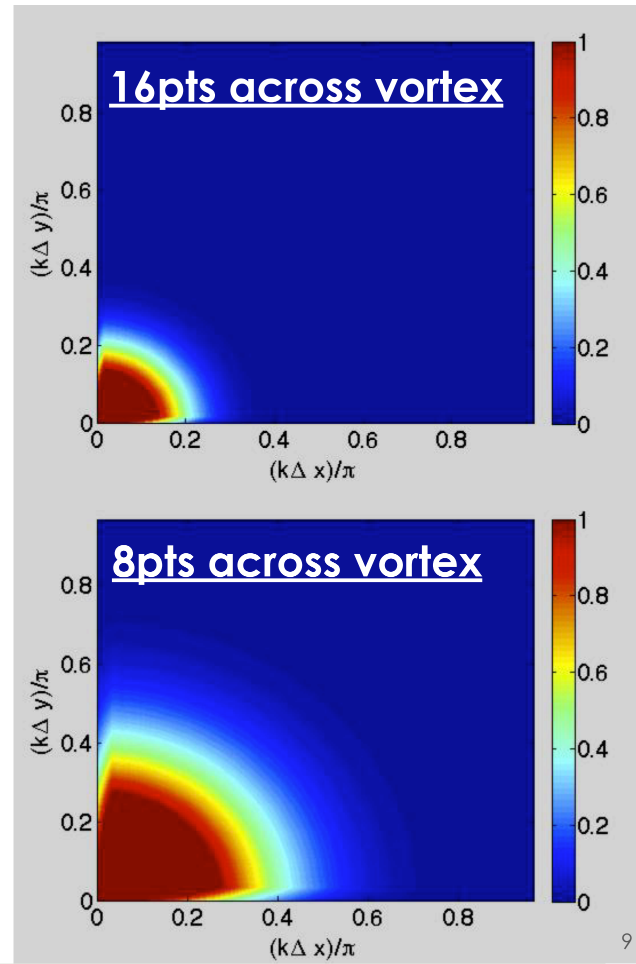
EF10(1.0)

$$Q = \left[1 + \left(\frac{\Delta x}{2} \right)^{10} \frac{\partial^{10}}{\partial x^{10}} \right] Q^{*}$$

$CFL_{u,1D} \approx 0.01 \qquad \alpha = 1, \phi = 1$

CD04 (11pt B&B)

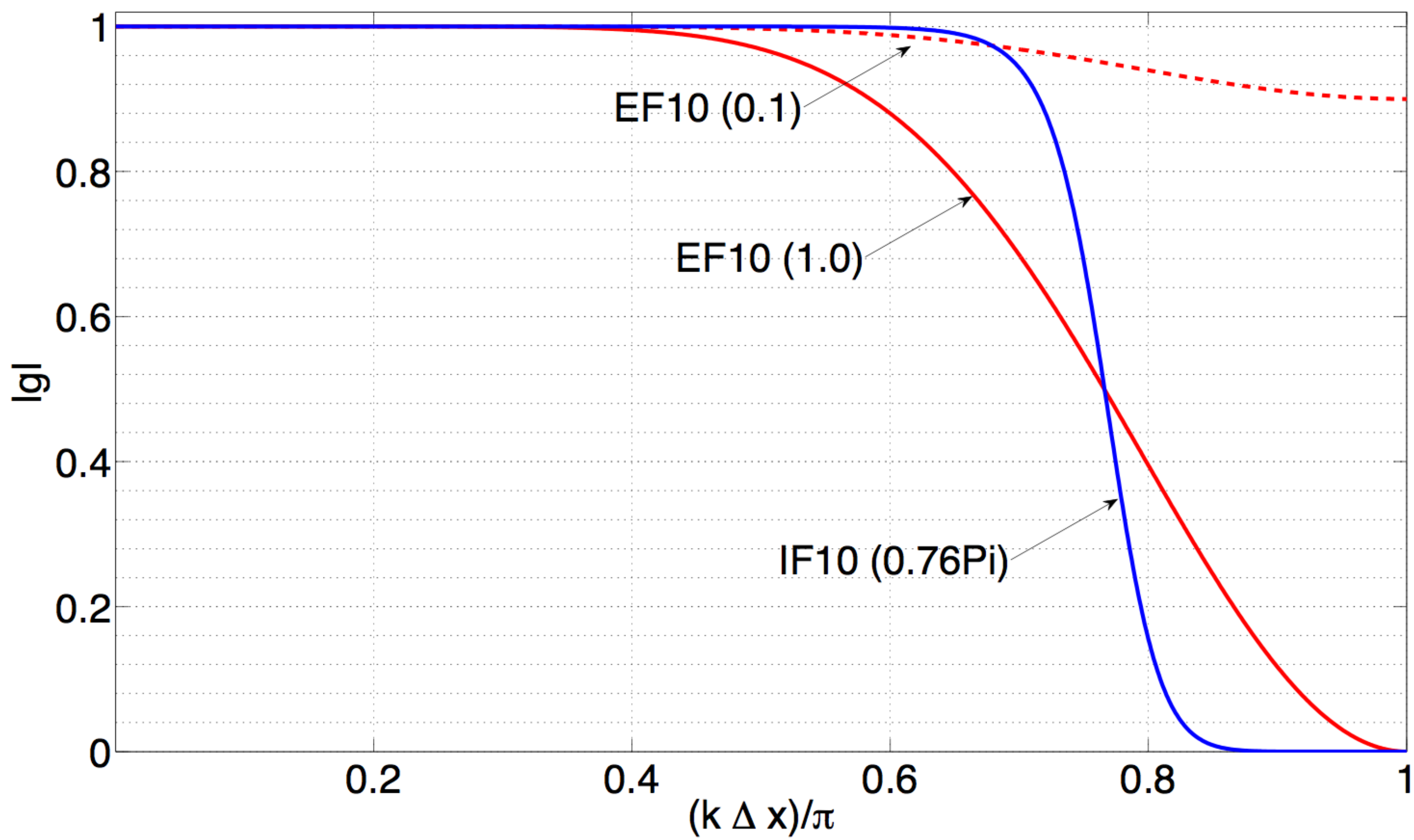




distance traveled: 500 widths

effectiveness of stabilization strategy dependent on spectral content

Damping Characteristics: Growth Factor



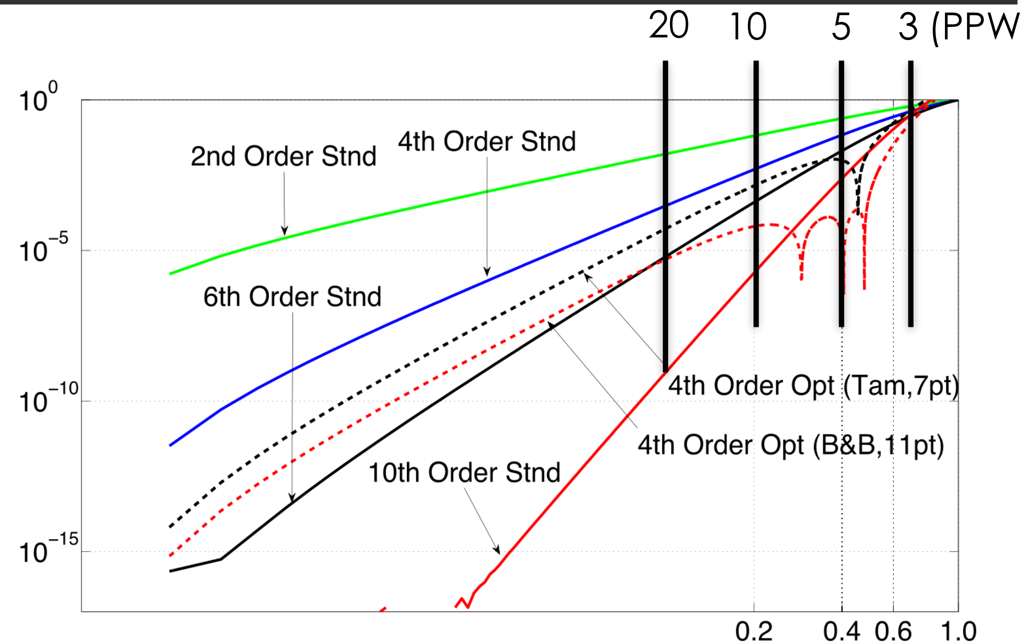
- need scale-discriminant, tunable formulations
- strong preservation of resolvable modes

Scale-Discriminant Stabilization

<ins>goal</ins>:
preserve accurately resolved frequencies and remove error-prone content

scale-discriminant dissipation

$$\text{error}_k = \frac{k - k_{\text{mod}}}{k} = 0.01$$



FD stencil	$(k\Delta x)_{cutoff}/\pi$	~PPW
CD02	***0.08***	***25***
CD04	0.24	9
CD06	***0.35***	***6***
CD10	0.46	5
CD04 (7pt Tam)	***0.48***	***5***
CD04 (11pt B&B)	0.55	4

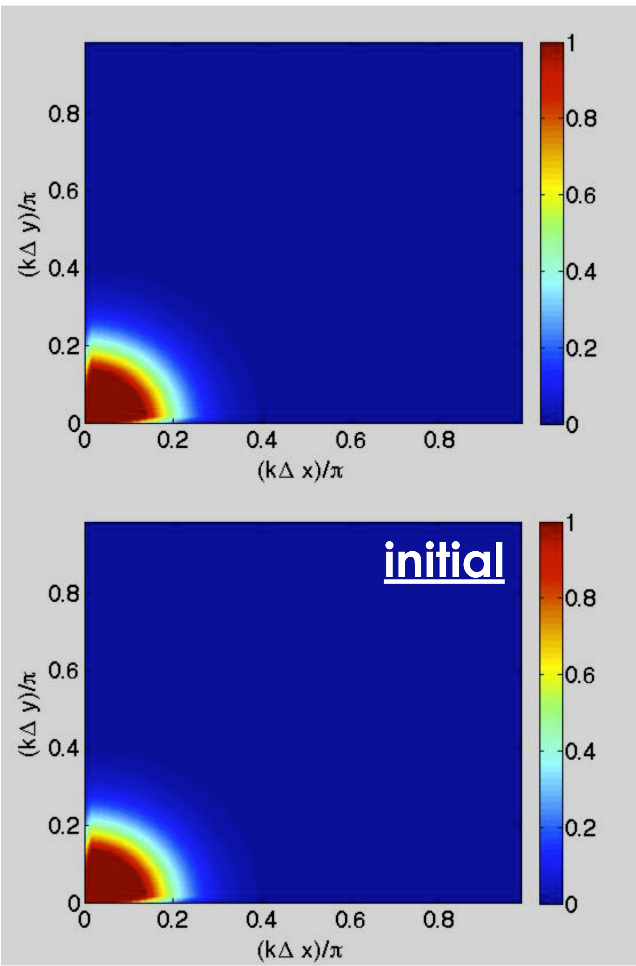
for the isentropic vortex…

- maintain coherence of vortex
- preserve vortex intensity

Isentropic Vortex: Scale-Discriminant Stabilization

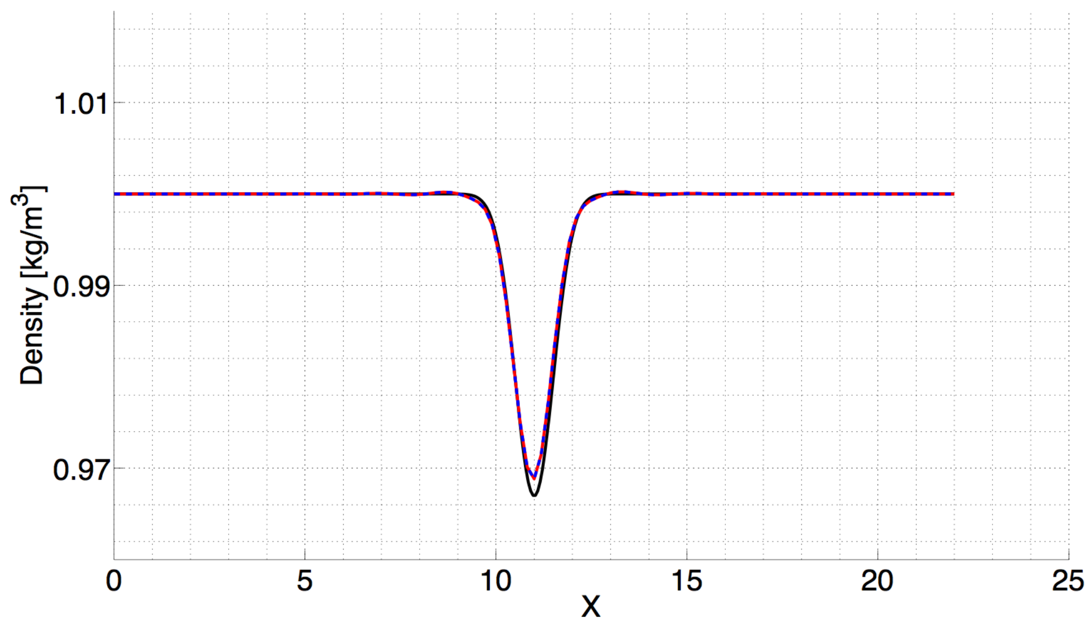
$CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1$

CD04 (11pt B&B) + IF10 (2/3Pi)



__ ref
__ IF10 (2/3Pi)
__ EF10

16 points across vortex



distance traveled: 500widths

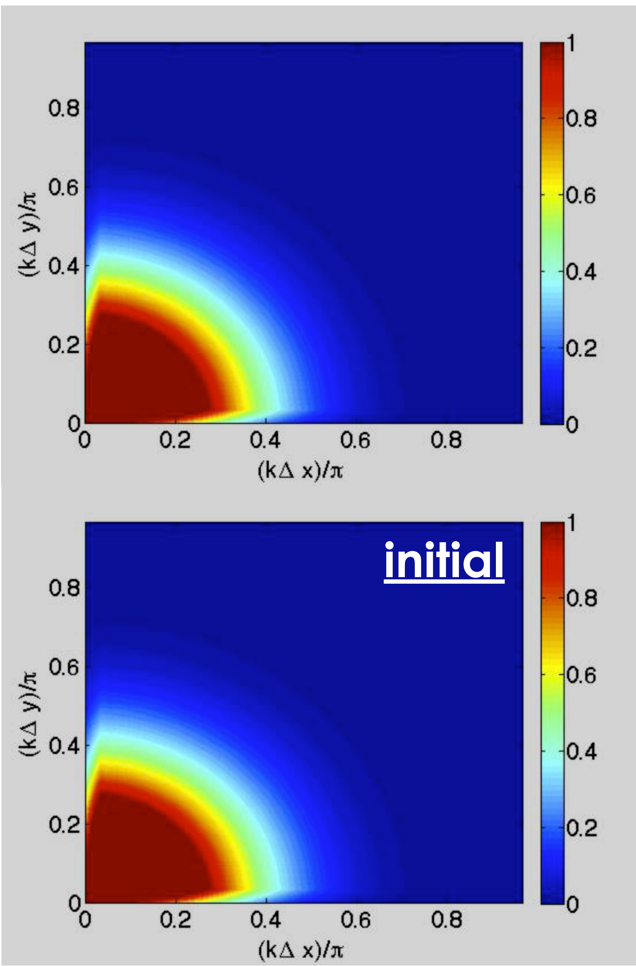
- traditional dissipation is sufficient when spectrum is well resolved

Isentropic Vortex: Scale-Discriminant Stabilization

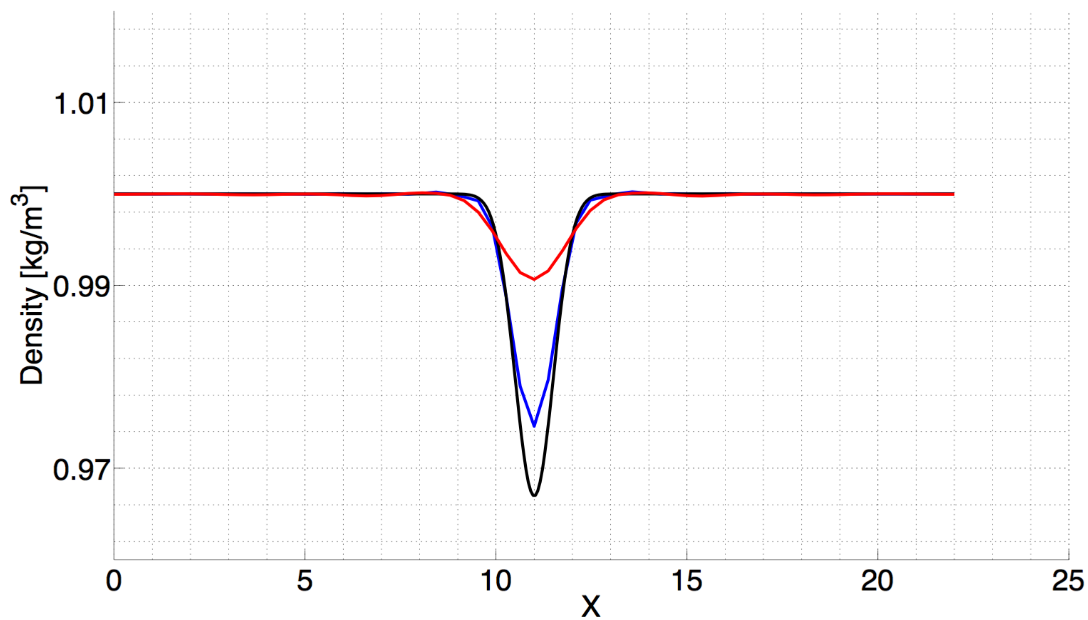
$CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1$

CD04 (11pt B&B) + IF10 (2/3Pi)

__ ref
__ IF10 (2/3Pi)
__ EF10



8 points across vortex



distance traveled: 500 widths

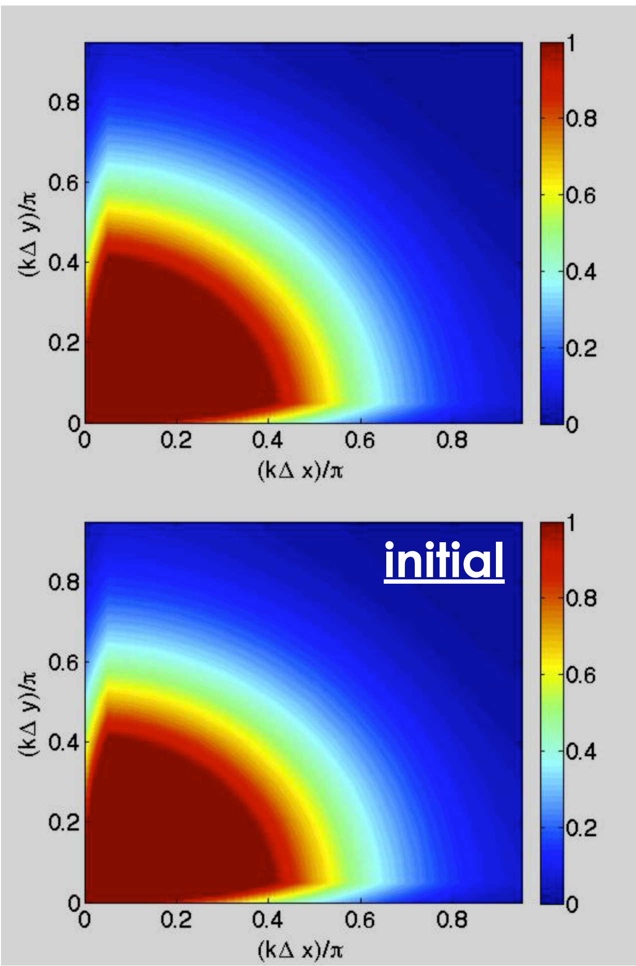
- scale-discriminant dissipation preserves structure
- robustness requires tuning to scheme resolvability
- efficacy limited by dissipation scheme

Isentropic Vortex: Scale-Discriminant Stabilization

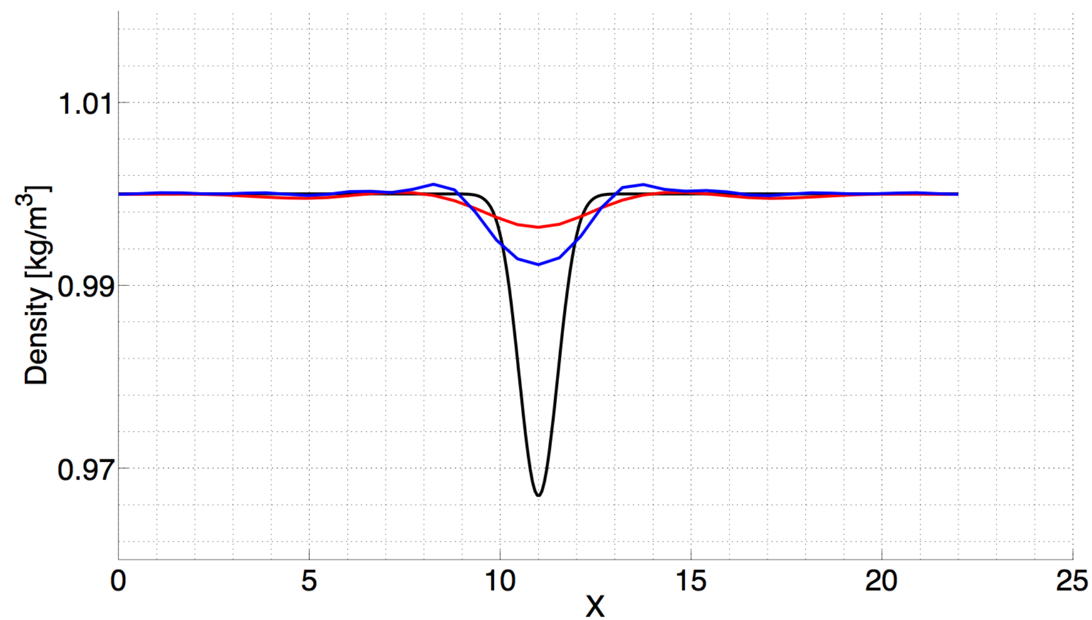
$CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1$

CD04 (11pt B&B) + IF48 (2/3Pi)

__ **ref**
__ **IF48 (2/3Pi)**
__ **EF10**



4 points across vortex



distance traveled: 500widths

- scale-discriminant dissipation preserves structure
- robustness requires tuning to scheme resolvability
- efficacy limited by dissipation scheme

Conclusions/Going Forward

- non-linear instabilities instigated by high frequency error (i.e.: dispersion, aliasing)
- remove error with respect to overall scheme resolvability – a robust strategy
- scale-discriminant dissipation provides stability while minimizing dissipation error

<u>going forward</u>:

- non-linear stability (residual filtering, skew-symmetric forms etc...)
- incorporating temporal error
- applications to explicit LES